On the Impact of Number Representation for High-Order LES

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Motivation

• LES is expensive…

• …really expensive.
Computer Arithmetic

• Binary floating point following IEEE 754

• \( x = \text{sign} \cdot \text{mantissa} \cdot 2^{\text{exponent}} \)

<table>
<thead>
<tr>
<th></th>
<th>sign</th>
<th>exponent</th>
<th>mantissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>binary32</td>
<td>1</td>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>binary64</td>
<td>1</td>
<td>11</td>
<td>52</td>
</tr>
</tbody>
</table>
Computer Arithmetic

• Complicated!

• If you think you understand floating point arithmetic—you don’t!
# Why Number Precision?

<table>
<thead>
<tr>
<th>Model</th>
<th>GB/s</th>
<th>Single</th>
<th>Double</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMD Radeon R9 Nano</td>
<td>512</td>
<td>8.19</td>
<td>0.51</td>
<td>16</td>
</tr>
<tr>
<td>AMD FirePro W9100</td>
<td>320</td>
<td>5.24</td>
<td>2.62</td>
<td>2</td>
</tr>
<tr>
<td>Intel Xeon E5-2699 v4</td>
<td>77</td>
<td>1.55</td>
<td>0.77</td>
<td>2</td>
</tr>
<tr>
<td>Intel Xeon Phi 7120A</td>
<td>352</td>
<td>2.42</td>
<td>1.21</td>
<td>2</td>
</tr>
<tr>
<td>NVIDIA Tesa K40c</td>
<td>288</td>
<td>4.29</td>
<td>1.43</td>
<td>3</td>
</tr>
<tr>
<td>NVIDIA Tesa M40</td>
<td>288</td>
<td>7.00</td>
<td>0.21</td>
<td>32</td>
</tr>
</tbody>
</table>
Potential Speedups

• If a code region is limited by…

  • FLOPs = $2x$ to $32x$

  • Memory bandwidth = $2x$

  • Disk I/O = $2x$

  • Latency (memory, disk, network, …) = $1x$
The Status Quo

• Extensive research in bars indicates that, if given the choice between a single and a double measure, the double wins every time.

• CFD codes are no exception.
Do We Need Double Precision?

- Very little research in the CFD space.

- Results mostly limited to steady state computations where double precision does appear to be necessary.
Methodology

• Rerun several of our *previous published* test cases using single precision arithmetic.

• Compare the results and assess the performance.
Experiments

• Using PyFR we have evaluated several *unsteady viscous test cases*.
  • Taylor–Green vortices.
  • Flow over a circular cylinder.
  • Flow over a NACA 0021.
3D Taylor–Green Vortex

- Standard test case for DG.
3D Taylor–Green Vortex

- Four **structured grids** with roughly **constant DOF count**.

<table>
<thead>
<tr>
<th>Order</th>
<th>$N_E$</th>
<th>$\Sigma N_u$</th>
<th>Memory / GiB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi = 2$</td>
<td>$86^3$</td>
<td>$258^3$</td>
<td>6.4</td>
</tr>
<tr>
<td>$\varphi = 3$</td>
<td>$64^3$</td>
<td>$256^3$</td>
<td>5.4</td>
</tr>
<tr>
<td>$\varphi = 4$</td>
<td>$52^3$</td>
<td>$260^3$</td>
<td>5.1</td>
</tr>
<tr>
<td>$\varphi = 5$</td>
<td>$43^3$</td>
<td>$258^3$</td>
<td>4.6</td>
</tr>
</tbody>
</table>
3D Taylor–Green Vortex

• Consider kinetic energy decay rate.

• Compare with van Rees et al.

• No difference between single and double.
3D Taylor–Green Vortex

- Performance on a two NVIDIA K40c's with GiMMiK.

<table>
<thead>
<tr>
<th>Order</th>
<th>GFLOP / stage</th>
<th>$t_w/ \sum N_u / 10^{-9} \text{s}$</th>
<th>GFLOP / s</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi = 2$</td>
<td>$1.84 \times 10^1$</td>
<td>4.8</td>
<td>8.9</td>
<td>222.1</td>
</tr>
<tr>
<td>$\varphi = 3$</td>
<td>$1.82 \times 10^1$</td>
<td>4.2</td>
<td>7.9</td>
<td>252.3</td>
</tr>
<tr>
<td>$\varphi = 4$</td>
<td>$1.92 \times 10^1$</td>
<td>4.4</td>
<td>8.6</td>
<td>255.9</td>
</tr>
<tr>
<td>$\varphi = 5$</td>
<td>$1.96 \times 10^1$</td>
<td>4.5</td>
<td>13.1</td>
<td>250.8</td>
</tr>
</tbody>
</table>
Flow Over a Cylinder
Flow Over a Cylinder

- Cylinder at \( \text{Re} = 3900 \), and \( \text{Ma} = 0.2 \) with \( p = 4 \).

- Mixed prism/tet grid of span \( \pi D \).
Flow Over a Cylinder

- Pressure coefficient on the surface.
- Compare with Lehmkuhl et al.
Flow Over a Cylinder

- Performance on a single NVIDIA K40c with GiMMiK.
- Tet operator matrices are small and prisms sparse.
- Overall speedup of ~1.6.
- Simulation results in heavy indirection; thus experiences less of an improvement from single precision.
NACA 0021

• Flow over a NACA 0021 at 60 degree AoA.

• \( \text{Re} = 270,000 \) and \( \text{Ma} = 0.1 \).

• Compare with experimental results of Swalwell.
NACA 0021

• 206,528 hexahedral elements.

• Span is four times the chord.

• Fourth order solution polynomials with full anti-aliasing.
NACA 0021

- Performance on 16 NVIDIA K80’s (32 GPUs).
- All operators are dense.
- Near the limit of strong scaling.
- Overall speedup of \(~1.8\).
Remarks and Closing Thoughts

For LES single precision is sufficient.